



**NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION**

**DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION**

**MASTER OF SCIENCE EDUCATION**

**ADVANCED NUMERICAL METHODS**

**PST 6132**

**Main Examination Paper**

**November 2024**

This examination paper consists of 3 pages

**Time Allowed: 3 hours**

**Total Marks: 100**

**Special Requirements: None**

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**External Examiner: Dr Sunzuma G**

**INSTRUCTIONS**

1. This paper consists of **SIX** questions. Answer **at most FIVE** questions.
2. All answers should be presented in good style.
3. Begin each full question on a new page.

**MARK ALLOCATION**

<b>QUESTION</b>	<b>MARKS</b>
<b>1.</b>	<b>20</b>
<b>2.</b>	<b>20</b>
<b>3.</b>	<b>20</b>
<b>4.</b>	<b>20</b>
<b>5.</b>	<b>20</b>
<b>6.</b>	<b>20</b>

**Q1.** Given the equation  $\frac{dy}{dx} = y + x$ , such that  $y(0) = 1$ .

- a) Use Picard's process of successive approximations, to obtain a solution up to the fifth approximation. [7]
- b) Use Taylor's method of third order to solve the equation. Hence find the values of  $y$  at  $x = 0.1$  and  $x = 0.2$ . [7]
- c) Check your answers in (i) and (ii) by finding the exact solution. [6]

**Q2.** a) i) Use the Euler's method to find the approximate value of  $y(1.4)$  for the given initial-value problem  $\frac{1}{x}y' - y^2 = 0$ ,  $y(1) = 1$  with  $n = 2$ . [7]

ii) Compare your approximate solutions with the exact solution

$$y(x) = \frac{2}{3-x^2}. \quad [3]$$

b) i) Use Runge-Kutta method of order two (Modified Euler's method) to find the approximate value of  $y(0.6)$  for the initial-value problem

$$y' = xy + x, \quad y(0) = 0, \quad \text{with } h = 0.2. \quad [7]$$

ii) Compare your approximate solution with the exact solution

$$y(x) = -1 + e^{x^2/2}. \quad [3]$$

**Q3.** a) Let  $f(x) = x - \cos x$

i) Apply two iterations of the Bisection method to  $f(x) = 0$ . Start with  $x^{(0)} = 0$  and  $x^{(1)} = 1$ . [3]

ii) Apply two iterations of the Newton-Raphson method to  $f(x) = 0$ . Start with  $x^{(0)} = 0$ . [3]

iii) Suppose you have to find the root of  $f(x) = 0$  to six decimal places. Would you use the Newton-Raphson or Bisection method? Justify your answer. [2]

b) Given  $\frac{dy}{dx} = x^2(1 + y)$  and  $y(1) = 1$ ,  $y(1.1) = 1.233$ ,  $y(1.2) = 1.548$ ,  $y(1.3) = 1.979$ , evaluate  $y(1.4)$  by Adams-Bashford method. [12]

**Q4.** a) Evaluate the following integral with number of sub intervals  $n = 6$ .

$$I = \int_0^{0.6} \frac{dx}{\sqrt{1+x}}.$$

i) Using Trapezoidal rule [3]

ii) Using Simpson's rule [3]

iii) Using Simpson's  $\frac{3}{8}$ th rule. [4]

b) Using Milne's method, find  $y(4.5)$  given  $5xy' + y^2 - 2 = 0$  given  
 $y(4) = 1, y(4.1) = 1.0049, y(4.2) = 1.0097, y(4.3) = 1.0143,$   
 $y(4.4) = 1.0187.$  [10]

**Q5.** a) Using quadratic Lagrange interpolation find the Lagrange interpolating polynomial  $P_2(x)$  and hence find value of  $y$  at  $x = 2$ . Given  $y(0) = 15,$   
 $y(1) = 48, y(5) = 85.$  [5]

b) Find the Newton's interpolating polynomial and also find the value of  $y$  at  $x = 5$ .

$x$	0	10	20	30	40
$y$	7	18	32	48	85

[6]

c) Find the eigenvalues and eigenvectors of the matrix  $\begin{pmatrix} 6 & 3 & 1 \\ 3 & 2 & 0 \\ 1 & 4 & 5 \end{pmatrix}$  by  
Power method taking  $x_0 = [1 \ 1 \ 1]^T.$  [9]

**Q6.** a) Solve the differential equations  $\frac{dy}{dx} = 1 + xz, \frac{dz}{dx} = -xy$  for  $x = 0.3$  using the fourth order Runge-Kutta method. Initial values are  $x = 0, y = 0, z = 1.$  [10]

b) Find the value of  $y(1.1)$  and  $y(1.2)$  from  $y'' + y^2y' = x^3; y(1) = 1, y'(1) = 1$  using the Taylor series method. [10]

**END OF QUESTION PAPER.**