NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

MASTER OF SCIENCE EDUCATION IN PHYSICS

MATHEMATICAL METHODS IN PHYSICS (PST 6170)

Main Examination Paper

November 2024

This Examination Paper consists of 4 printed pages

Time allowed	: 3 hours
Total Marks	: 100
Special requirements	: None
Internal Examiner	: J. Hlongwane
External Examiner	: Dr N Zezekwa

INSTRUCTIONS

1. Answer any **four** (4) questions. Each question carries 25 Marks.
2. Show all your working steps clearly in any calculation.
3. Start the answer for any question on a new page.

MARK ALLOCATION

QUESTION	MARKS
1.	25
2.	25
3.	25
4.	25
5.	25
TOTAL (Four questions)	100

Fourier series: Suppose f is a periodic function with a period $T = 2L$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

Euler- Fourier formulas:

$$a_m = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{m\pi x}{L}\,dx\,, m = 0,1,2,3,4\ldots$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx\,, n = 1,2,3,4\ldots$$

1. a. Two complex numbers are $z_1 = 8 + 3i \text{ and } z_2 = 9 - 2i$ Calculate: [4]

   (i) $\frac{z_1}{z_2}$ (ii) $Z_1 Z_2$

   b. Solve the first order partial differential equation by separation of variables. [5]

$$\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = 0$$

   c. 3 vectors A,B, C are denoted by: [8]

$$A = 3\hat{x} - 2\hat{y} + 2\hat{z}$$

$$B = 6\hat{x} + 4\hat{y} - 2\hat{z}$$

$$C = -3\hat{x} - 2\hat{y} - 4\hat{z}$$

   Compute the values of $A \cdot B \times C$ and $A \times (B \times C)$

   d. Discuss the application of any two mathematical methods in the use of renewable energy. [8]

2. a. Use Maxwell's equation in a vacuum to derive the electromagnetic wave equation in terms of the electric field. [9]

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \cdot \mathbf{E} = 0,$$

$$\nabla \times \mathbf{B} = \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t},$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

   b. From 2(a) deduce the speed of light. [3]

   c. Outline how light energy from the sun can be used to produce green energy. [10]

   d. Explain the physical significance of the mathematical curl equation of the electric field. [3]

3. a. Use at least two examples to explain the application of triple integrals in physics. [4]

b. Compute the solution of : [6]

$$\int_{z=0}^{z=1}\int_{y=2}^{y=4}\int_{x=-1}^{x=5}(x+yz^2)dx\,dy\,dz$$

b. Apply the method of determinants to solve this system of simultaneous equations. [6]

$$-x+y+2z=2$$
$$3x-y+z=6$$
$$-x+3y+4z=4$$

c. Use the power series method to solve the following differential equation clearly showing your working. [9]

$y'' + y' = 0$

4. a. Find the Fourier coefficients and Fourier series of the **square-wave function** defined by [9]

$$f(x) = \begin{cases} 0 & \text{if } -\pi \leqslant x < 0 \\ 1 & \text{if } 0 \leqslant x < \pi \end{cases} \quad \text{and} \quad f(x+2\pi) = f(x)|$$

b. Compute the Wronskian and comment on the result. [6]

$$\begin{vmatrix} e^x & e^{-x} & \cosh x \\ e^x & -e^{-x} & \sinh x \\ e^x & e^{-x} & \cosh x \end{vmatrix}$$

c. Use any appropriate mathematical method to solve this system of equations: [4]

$$4x_1 + 3x_2 = 12$$
$$2x_1 + 5x_2 = -8$$

d. A simple harmonic oscillator of mass m and natural frequency ω_0 experiences an oscillating driving force given by

$$f(t) = ma\cos\omega t$$

(i) Show that its equation of motion is : [3]

$$\frac{d^2x}{dt^2} + \omega_0{}^2 = a\cos\omega t$$

Where x is the position. Given that at t=0, $\frac{dx}{dt} = 0$

(ii) Find the function x(t) and describe the solution if omega is approximately but not exactly equal to ω_0. [3]

5. a. Given that the trigonometric functions and hyperbolic functions follow the following power series : [10]

$$\sin z = \sum_{n=1,\text{odd}}^{\infty} (-1)^{(n-1)/2} \frac{z^n}{n!} = \sum_{s=0}^{\infty} (-1)^s \frac{z^{2s+1}}{(2s+1)!},$$

$$\cos z = \sum_{n=0,\text{even}}^{\infty} (-1)^{n/2} \frac{z^n}{n!} = \sum_{s=0}^{\infty} (-1)^s \frac{z^{2s}}{(2s)!},$$

$$\sinh z = \sum_{n=1,\text{odd}}^{\infty} \frac{z^n}{n!} = \sum_{s=0}^{\infty} \frac{z^{2s+1}}{(2s+1)!},$$

$$\cosh z = \sum_{n=0,\text{even}}^{\infty} \frac{z^n}{n!} = \sum_{s=0}^{\infty} \frac{z^{2s}}{(2s)!}.$$

Show that $i \sin z = \sinh iz$ and $\cos iz = \cosh z$

b. The population of radioactive isotopes reduces such that their number N (t) present at any given time t is directly proportional to their rate of decrease. Design a mathematical model to express N (t) in terms of the initial number of atoms N_0 present at t = 0. [6]

c. Solve the integral $\int_0^1 \int_x^{2x} (2 + x^2 + y^2)\, dy\, dx$ [6]

d. Outline one application of the Cauchy- Schwartz inequality in Physics. [3]

END OF EXAMINATION