NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

# COMPUTATIONAL DISCRETE MATHEMATICS

**PST 6175/ PTE 6175**

**MAIN EXAMINATION PAPER**

**NOVEMBER 2024**

This Examination Paper consists of 4 pages

**Time Allowed:** 3 hours

**Total Marks:** 100

**Special Requirements:** NONE

**Internal Examiner:** Ms D Moyo

**External Examiner:** Dr B Moyo

## INSTRUCTIONS

1. Answer any four (4) questions.
2. Each question carries 25 marks

## MARK ALLOCATION

| QUESTION | MARKS |
|---|---|
| 1. | 25 |
| 2. | 25 |
| 3. | 25 |
| 4. | 25 |
| 5. | 25 |
| TOTAL | 100 |

## QUESTION 1

a) Given the functions $f(x), g(x) and\ h(x)$.

$f(x) = x^2$-3, $g(x) = 2x + 1, h(x) = \frac{x}{2}$

Find;

i. $fg(x)$ [3marks]

ii. $gh(x)$ [3marks]

iii. $h^{-1}$ [3marks]

b) Construct a truth table to show that this is a tautology.

$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \rightarrow p))$ [6marks]

c) Using the relationships between sets and sentential functions and their properties, demonstrate that:

(A∩B) U(A∩C) =A∩(BUC) [5marks]

d) Given $P(x): x > 3$.What is the truth value of $\exists x P(x)$ in the domain of real numbers. [5marks]

## QUESTION 2

a) Using Euclidean algorithm, calculate the $\gcd(1424,3084)$ [4marks]

b) Find all integers *x* and *y* such that $258x + 147y = 369$. [8marks]

c) Let $N = \{1,2,3 \ldots.\}$. Show that the R= $\{$(x, $y|x - y)\}$ is divisible by 2 for every $x\ and\ y$ and $y \in N$ is an equivalence relation. [7marks]

d) Given $f(x) = x^2 + 3x - 5$. Express $f(2x - 1)$ in the form

$ax^2 + bx + c$. [6marks]

## QUESTION 3

a) Expound on the following:

i. Tautology proposition [3marks]

ii. Contingency proposition [3marks]

iii. Contradiction proposition [2marks]

b) A box of jelly beans contains a mixture of 10 red, 18 blue, and 12 green jelly beans. Suppose that these are well-mixed and that the probability of pulling out any one jelly is the same.

i. What is the probability of randomly selecting two blue jelly beans from the box? [3marks]

ii. What is the probability of randomly selecting two beans that have similar colors? [4marks]

iii. What is the probability of randomly selecting two beans that have different colors? [4marks]

c) It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, the probability for a false positive (a non-spam email detected as spam) is 5%.

If an email is detected as spam. What is the that it is infact a non-spam email? [7marks]

## QUESTION 4

a) For any natural number $n$ and any real number $x$ prove that;$(1-x)(1+x+x^2 \ldots.+x^n) = 1-x^{n+1}$ [6marks]

b) Given that f(x)=x². Show that f(x) is not injective under real numbers. [3marks]

c) Given that f(x)=x² and g(x)=x+1. Find f(g(x)). [3marks]

d) Outline the step-by-step algorithm process used in graph isomorphism. [8marks]

e) Explain the concept of graph isomorphism. Then, describe how you would determine whether two given graphs G and H are isomorphic [5 marks]

**QUESTION 5**

a) Discuss using appropriate examples the applications of graph theory in 21$^{st}$ century. [10marks]

b) Explain how many edges a graph has if it has 5 vertices, each with a degree of 4. [5marks]

c) Figure 5.1 shows a directed graph whose vertices a, b, c, d and e.

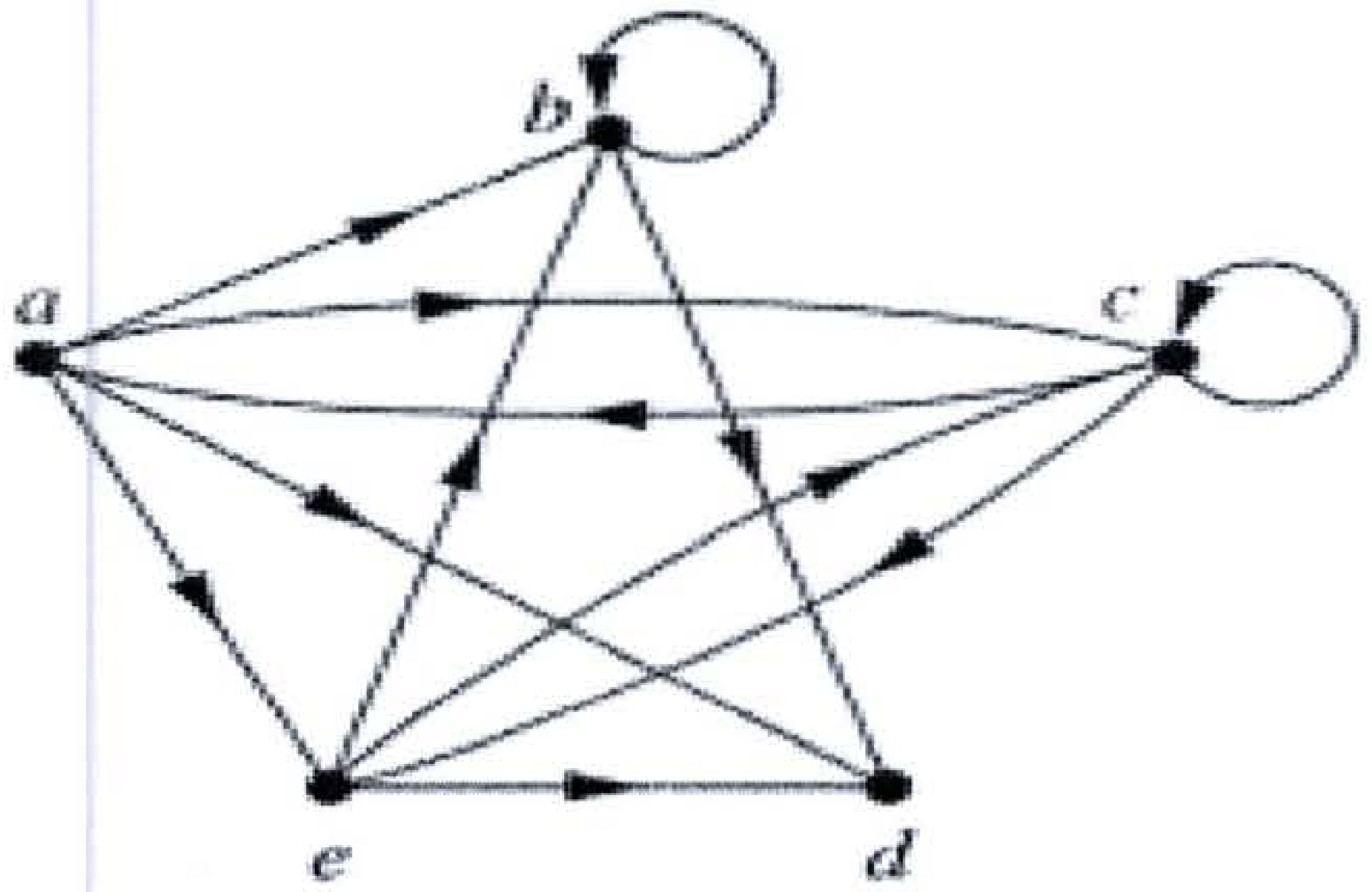


Figure 5.1 Directed graph.

Find the in-degree and out-degree of each vertex. [10marks]

**END OF QUESTION PAPER**