NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION
MASTER OF SCIENCE EDUCATION

NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS PST 6233

Main Examination Paper

November, 2024

This examination paper consists of 4 pages

Time Allowed: 3 hours
Total Marks: 100
Special Requirements: NONE

Examiner's Name: MOYO KUDA

External Examiner: Dr Sunzuma G

INSTRUCTIONS

1. This paper consists of **five** questions. Answer **four** questions.
2. All answers should be presented in good style.
3. Begin each full question on a new page.

MARK ALLOCATION

QUESTION	MARKS
1.	25
2.	25
3.	25
4.	25
5.	25
TOTAL	100

1. (a) Solve the Laplace equation $u_{xx} + u_{yy} = 0$ inside a square region bounded by the lines *x=0, x=4,y=0, y=4* given that $u = x^2y^2$ at the boundary using the relaxation method.

[15]

(b) Find the values of *u(x,t)* satisfying the parabolic equation $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ and the boundary

conditions $u(0,t) = 0 = u(8,t)$ and $u(x,0) = 4x - \frac{x^2}{2}$ at the points

$x = i: i = 0,1,2,....7$ and $t = \frac{1}{8}, j = 0,1,2,.....5.$

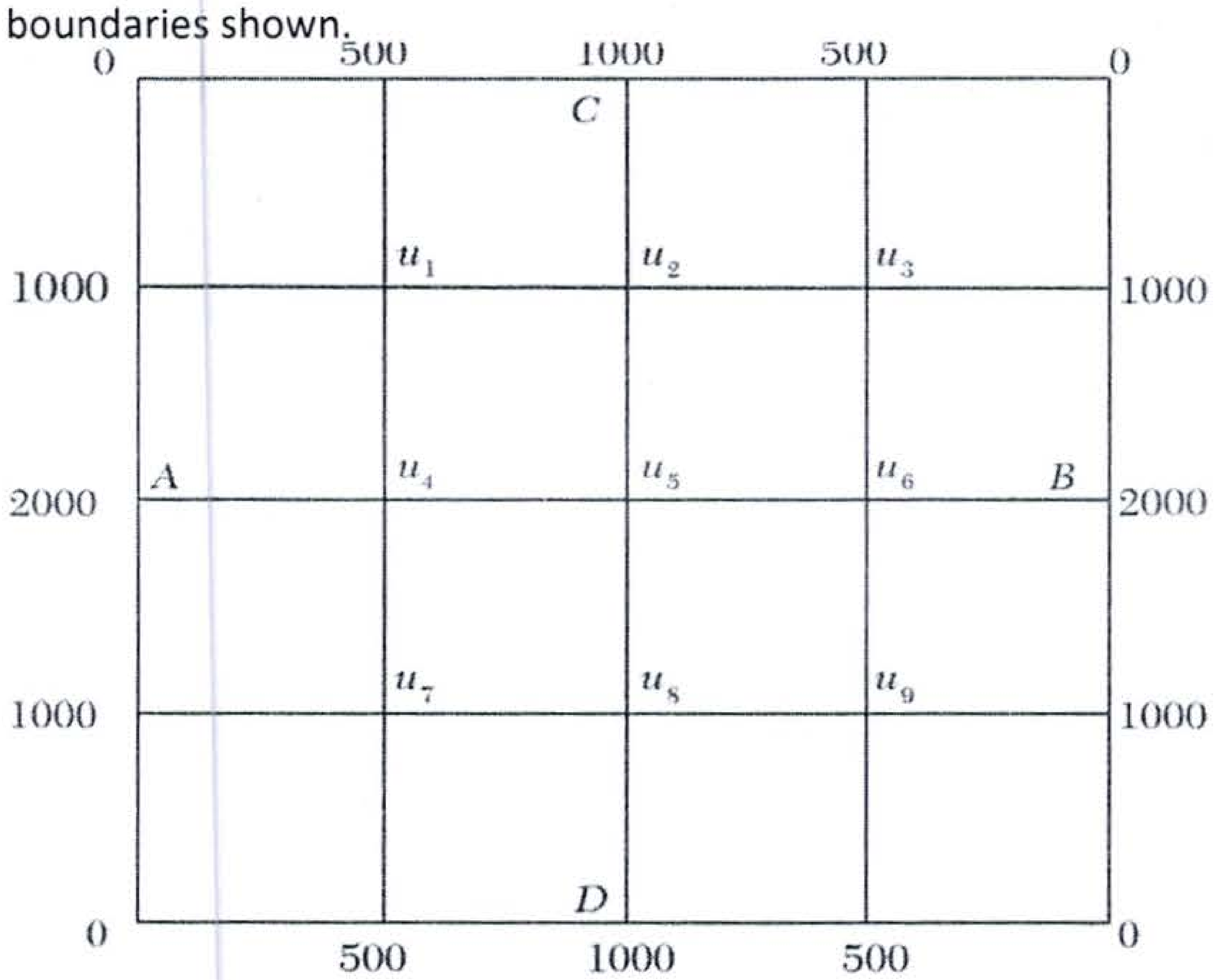
[10]

2. (a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in $0 \leq x \leq 5$, $t \geq 0$ given that $u(x,0) = 20, u(0,t) = 0, u(5,t) = 100$.
Compute *u* for the time step with *h=1* by the Crank- Nicholson method.
[10]

(b) Find the values of $u(x,t)$ satisfying the parabolic equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ by the Bender Schmidt method subject to the conditions;

$u(0,t) = 0, u(1,t) = 0, u(x,0) = \sin\pi x.$
[15]

3. (a) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the square mesh below and the boundaries shown.



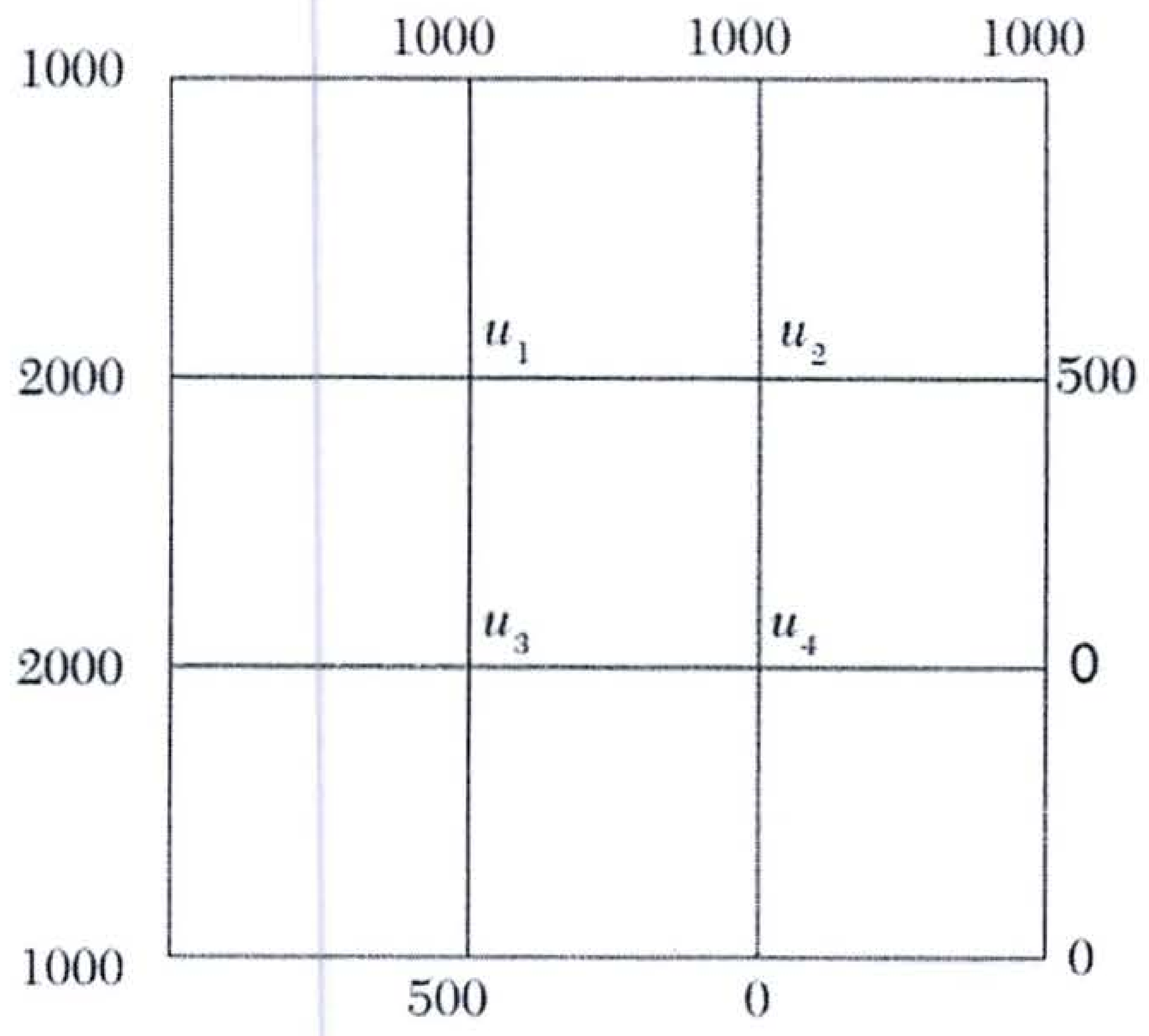
[15]

(b) Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$, taking $\Delta x = 1$ up to t=1.25. The boundary conditions are $u(0,t) = u(5,t) = 0, ut(x,0) = 0$ and $u(x,0) = x^2(5-x)$. [10]

4.(a)Given the values of *u(x,y)* on the boundary of the square mesh below, evaluate the function $u(x,y)$satisfying the Laplace equation$\nabla^2 u = 0$ at the pivotal points of the square mesh below by:

(i) Jacobi's method [15]

(ii) Gauss-Seidal method. [10]



5. (a) Solve $y_{tt} = y_{xx}$ up to $t = 0.5$ with a spacing of 0.1 subject to

$y(0,t) = 0, y(1,t) = 0, yt(x,0) = 0$ and $y(x,0) = 10 + x(1-x)$. [15]

(b) Classify the following equations:

(i) $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$ [5]

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + (1-y^2)\frac{\partial^2 u}{\partial y^2} = 0 \quad -\infty < x < \infty, -1 < y < 1$ [5]

END OF EXAMINATION PAPER