NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

MECHANICS 5 (PST 6271)

MASTER OF SCIENCE EDUCATION IN PHYSICS

Main Examination Paper

November 2024

This Examination Paper consists of 4 printed pages

Time allowed : 3 hours

Total Marks : 100

Special requirements : None

Internal Examiner : Mr J. Hlongwane

External Examiner : Dr Zezekwa

INSTRUCTIONS

1. Answer any four (4) questions. Each question carries 25 Marks.
2. Show all your working steps clearly in any calculation.
3. Start the answer for any question on a new page.

MARK ALLOCATION

QUESTION	MARKS
1.	25
2.	25
3.	25
4.	25
5.	25
TOTAL (4 QUESTIONS)	100

1. a. Suppose that the forces acting on a system of particles are derivable from a potential function V (suppose the system is conservative). Prove that if the Lagrangian $L = T - V$ then: [6]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_\alpha}\right) - \frac{\partial L}{\partial q_\alpha} = 0$$

b. Study Fig 1.1 and compute the Lagrangian for the simple pendulum system shown. [6]

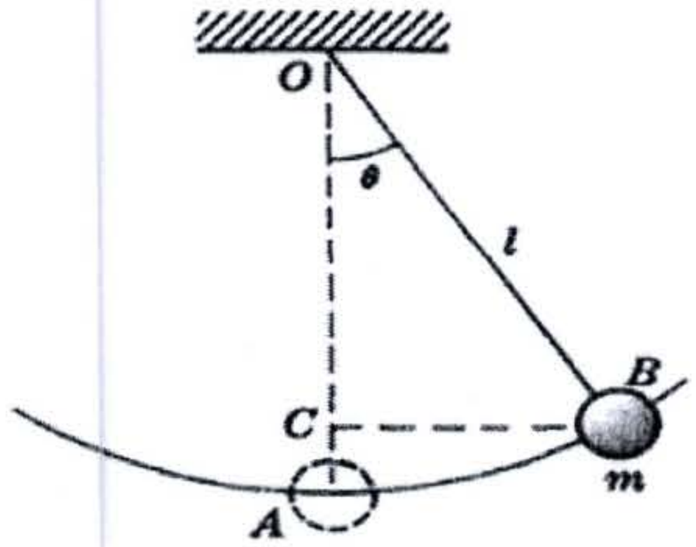


Fig 1.1

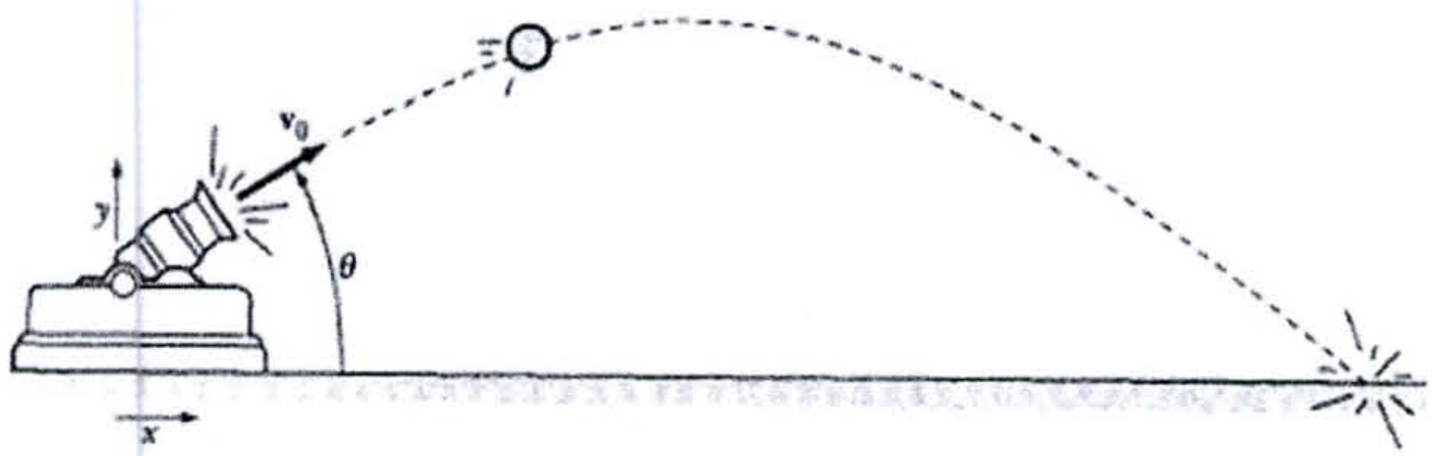
c. Use your answer to 1(b) to deduce the equation of motion for the system. [5]

d. Discuss four advantages of Lagrangian and Hamiltonian classical mechanics formulations over Newtonian mechanics. [8]

2. a. For holonomic constraints, use dot cancellation procedures to prove that: [8]

$$\frac{\partial \vec{r}_i}{\partial q_k} = \frac{\partial \dot{\vec{r}}_i}{\partial \dot{q}_k}$$

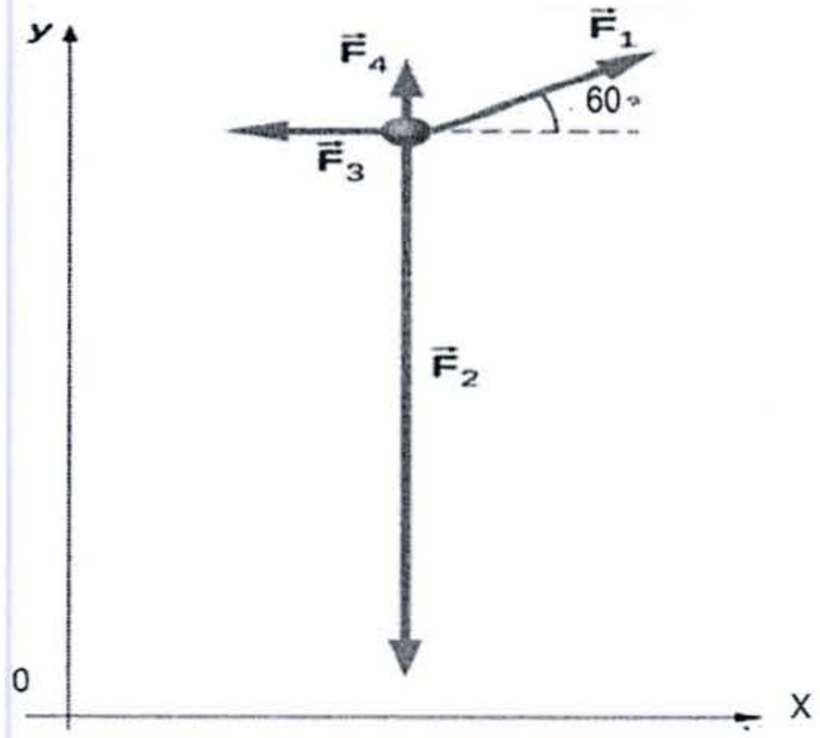
b. Fig 2.1 shows a big gun being fired at an angle of elevation θ, the bullet flies off as a projectile. Ignore the effects of air resistance. Let the muzzle velocity be V_0. [4]



Calculate as a function of time, the bullet's horizontal range.

c. Compute the bullet's velocity just before hitting the ground. [5]

d. A particle of mass m=5.3kg is acted upon by four forces as shown in Fig 8.1. Calculate the net acceleration of the particle. (You may apply D' Alembert's principle). [8]



3. a. Given that the Lagrangian of a harmonic oscillator is given by: [10]

$$L(q,\dot{q}) = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2$$

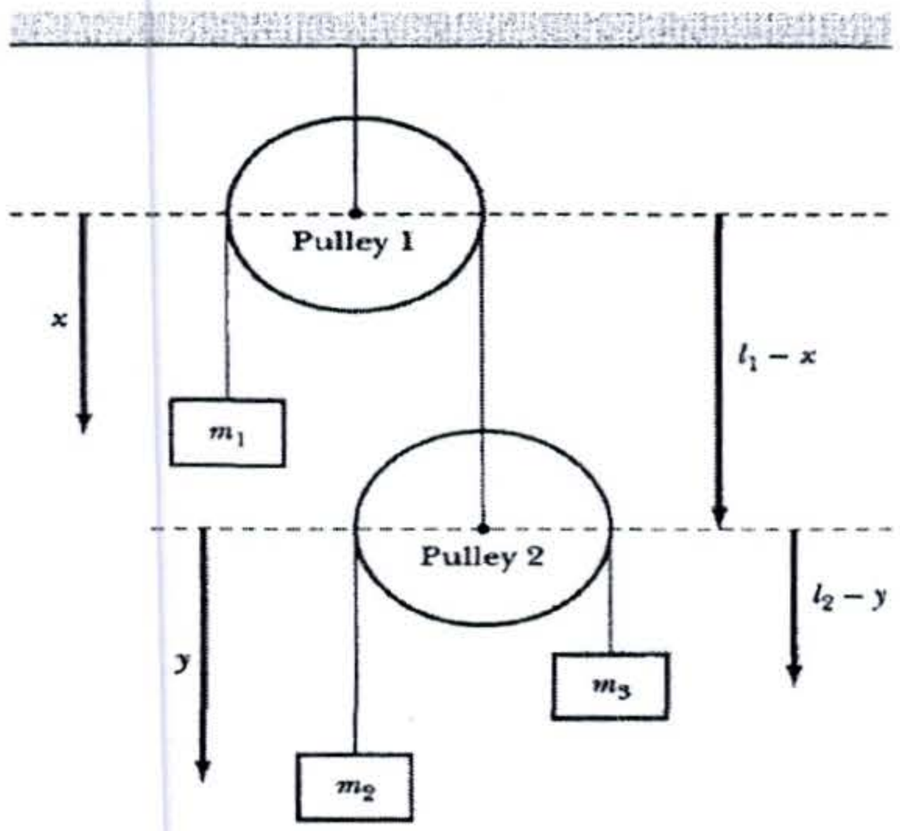
Compute the Hamiltonian of the system.

b. A system of two bodies acted upon by a central force, with no external forces can be reduced to a one-body system by equation $\bar{R} = \frac{m_1\bar{r}_1 + m_2\bar{r}_2}{m_1 + m_2}$ as three of our generalized coordinates. For the other three, we first use the Cartesian components of the relative coordinate $\bar{r} = \bar{r}_1 - \bar{r}_2$. The combined mass is given by $M = m_1 + m_2$

i. Express $\bar{r}_1$ and $\bar{r}_2$ in terms of $\bar{R}$ and M [8]

ii. Compute the kinetic energy of the system (you may use the reduced concept to simplify the final expression) [7]

4. a. Deduce the equation of motion for the double pulley system shown in Fig 4.1. Consider the pulleys to be massless and take l_1 and l_2 to be the lengths of the rope freely hanging from each pulley. X and y are distances measured from the centres of each pulley. [12]



b. Outline how the knowledge of mechanics you have gained in your studies can be applied to solve a named problem you have identified in your local community. [13]

5. a. Define the following terms: scleronomic, rheonomic, and holonomic constraints. [3]

b. Discuss five (5) situations in everyday life where mechanics principles are applied. [10]

c. Starving villagers in a remote rural area with a dense forest with very tall trees have trouble crossing a river infested with huge crocodiles to a neighbouring village with plenty of food. Apply your knowledge of Physics in general and mechanics in particular to design an environmentally friendly system that can assist them to cross the river to obtain food from their neighbours (you may draw illustrative diagrams). [12]

END OF EXAMINATION