

JULY/AUGUST 2004

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY TWO** questions from Sections B. Some probability functions and some formulae for Regression Analysis are given in APPENDIX 1 at the end of the question paper.[†]

SECTION A: Answer ALL questions in this section [60].

- A1. Consider the experiment of tossing a fair die. Denote by A and B the following events:
 $A = \{\text{Observing an even number of dots on the upper face of the die}\}$,
 $B = \{\text{Observing a number of dots less than or equal to 3 on the upper face of the die}\}$.
Find the probability of the event A , given the event B . [3]

- A2. Suppose that the random variable X is normally distributed with mean μ and variance σ^2 . Consider the transformation

$$Z = \frac{x - \mu}{\sigma}$$

Show that

- (a) $E(Z) = 0$, [3]
(b) $Var(Z) = 1$. [4]

- A3. Test for impurities commonly found in drinking water from private wells showed that 30% of all wells in a particular country have impurity A. If a random sample of 5 wells is selected from the large number of wells in the country, what is the probability that:

- (a) Exactly 3 will have impurity A? [2]
 (b) At least 3 will have impurity A? [4]
 (c) Fewer than 3 will have impurity A? [4]

A4. The mean and standard deviation of n measurements randomly sampled from a normally distributed population are 33 and 4, respectively. Construct a 95% confidence interval for the population mean (μ) when

- (a) $n = 5$, [5]
 (b) $n = 33$. [5]

A5. A random sample of size 150 is selected from a normal population and the number of successes is 60.

- (a) Find \hat{p} the sample proportion of successes. [2]
 (b) Construct a 90% confidence interval for the population proportion p . [8]

A6. Suppose that we are investigating the safety of a dangerous intersection. Past police records indicate a mean of 5 accidents per month at this intersection. Suppose the number of accidents is distributed according to a Poisson distribution.

- (a) Calculate the probability in any month of less than 5 accidents. [6]
 (b) Calculate the mean and variance of the number of accidents in 10 days. [4]

A7. The ages of a sample of the people attending a training course on networking in Hanoi are:

29 20 23 22 30 32 28 23 24 27 28 31 32 33 31 28 26 25 24 23 22 26 28 31 25 28 27 34
 Compute the mean and the standard deviation of the raw data set. [2,8]

SECTION B: Answer TWO questions in this section [40].

B8. A random sample of n observation is selected from a population with unknown mean μ and variance σ^2 . For each of the following situations, specify the test statistic and rejection region.

- (a) $H_0 : \mu = 40$, $H_1 : \mu > 40$; with $n = 35$, $\bar{x} = 60$, $s^2 = 64$ and $\alpha = 0.05$ [7]
 (b) $H_0 : \mu = 120$, $H_1 : \mu \neq 120$; with $n = 40$, $\bar{x} = 140.5$, $s = 9.6$ and $\alpha = 0.01$ [7]

